

POLISH SCHOOL OF RELIABILITY AND SAFETY

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Abstract

The history beginning of School of Safety and Reliability in Poland is represented in paper. Authors of the School are among other Prof. Krystyna Ważyńska-Fiok, Prof. Jerzy Jaźwiński, Prof. Jan Borgoń, Prof. Zbigniew Smalko, Prof. Józef Żurek, Dr. Tadeusz Salamonowicz, Prof. Tadeusz Szopa, Prof. Alfred Brandowski, Dr. Krzysztof Kołowrocki. Among scientific institutions having greatest contributions in development of the Polish School of Safety and the Reliability are Committee of Transport and Committee of Machines Construction of Polish Academy of Sciences, Air Force Institute of Technology, Faculty of Transport of Warsaw University of Technology, Polish Cybernetic Society, Polish Society of Safety and Reliability.

Conferences "Problems of the Reliability in the Transport" and "Safety of Systems" "International Conference of Safety and Reliability KONBIN" determine leading in the country and recognized in the world meeting points scientist occupying with these scientific problems. The reliability and the safety are with themselves related.

The elements the three-stage, renewable three-stage element, the fourth state of security threat, the fifth state of feeling security threat are an object of the paper. Markov processes and the theory tracing systems of the safety are presented in paper. The three-stage object non-renewable, the model of the object of damages, the diagram presenting time of flight, the elementary model system, "the system protected - the system switching on - the system protecting", the object of the structure ranker complex of three-stage elements the model of the three-stage renewable object with reversible state of efficiency deceptive, the reduced model, the three-stage renewable object are discussed in the paper.

Keywords: *transport systems, reliability, safety, Markov processes, renewable object*

1. Introduction

In 1976 Prof. Krystyna Ważyńska-Fiok organized at the Transport Institute of Warsaw University of Technology a seminar "Reliability in Transport". Seminar works were summarized at conferences organized periodically in association with Prof. Jerzy Jaźwiński and sponsored by the Transport Committee of Polish Academy of Sciences, the Polish Academy of Sciences – Machine Design Committee, Air Force Institute of Technology and Transport Faculty of the Warsaw University of Technology. The conferences and symposia took place in 1980, 1983, 1986, 1990, 1993, and 1997

In 1985 Prof. Krystyna Ważyńska-Fiok in association with Prof. Jerzy Jaźwiński established a Team for System Safety at the Polish Cybernetics Society which was organizing seminars on "System Safety". Seminar works were summarized at conferences organized periodically and sponsored by the Transport Committee of Polish Academy of Sciences, Air Force Institute of Technology and Transport Faculty of the Warsaw University of Technology and Polish Cybernetics Society.

Conferences "Problems of Safety in Transport" and "System Safety" were connected mainly with such problems in transport. The Polish Society of Safety and Reliability was founded in 1998 due to cooperation with Prof. Jerzy Jaźwiński, Prof. Zbigniew Smalko, Prof. Józef Żurek, Dr. Tadeusz Salamonowicz, Prof. Tadeusz Szopa, Prof. Alfred Brandowski, and Dr. Krzysztof Kołowrocki. The society has undertaken it to integrate these two main streams of scientific activity

into one and to broaden activities by adding the activity with the subject of safety and reliability within a system "man – technology – environment". Thus emerged the KONBIN "International Conference of Safety and Reliability". The KONBIN organizes cyclic conferences. These conferences took place in November 1999 in Kościelisko - Poland, in May 2002 in Szczyrk – Poland, in May in Gdynia – Poland and in May 2005 in Krakow.

From the activities dealing in reliability and safety emerged a team under the direction of Prof. Kristina Ważyńska-Fiok and Prof. Jerzy Jaźwiński. The scientists have made an observation that reliability and safety are interrelated. Thus a three-state element was conceived in which a state of full suitability S_0 , a state of efficiency unreliability S_S and a state of safety unreliability S_B were singled out. In the course of further analysis a renewable three-state element was discussed. It was assumed the state of efficiency unreliability could be reversible. As the concept was further developed a fourth state was introduced, which received a status of a state of a threat to safety S_{ZB} . This is a transient state between the state of a full suitability and the sated of safety unreliability.

During this state a dangerous situation can be counteracted i.e. the state of an endangered safety can be reversed. Authors attempted to discuss a fifth state i.e. a state of a feeling of endangered safety S_{pZB} . Although this state is irrational it still can cause a real threat. To model this system authors used Markov's processes. The analysis of these models was developed by a number of teams. This analysis was illustrated in Professor's assistant qualification thesis works of Prof. Borgoń's and Prof. Józef Żurek's. In the era of terrorist attacks a theory of follow-up safety systems was created. Prof. Smalko is the author of this concept.

2. Three-State Unrenewable Object

1. In an object damages can occur, which can cause different results:
 - In an electric device/system – "object" - damage like a break can occur, which cuts the power supply – efficiency unreliability. Damage like short-circuit can result in a fire – safety unreliability. This example points to various damages resulting in various effects.
 - A system consists of a functional device, e.g. an electric stove, or a radio-station and an air-condition device (cooling). Damage to the functional device causes efficiency unreliability, an operation break of a device. A damage to the air-condition (cooling) apparatus cause a safety unreliability, e.g. a fire.
2. In an object damages occur the effects of which depend on the circumstances of an occurrence and ways of counteracting it.
 - A damaged plane navigation system during daylight makes it impossible to land at an air base, when basing on observation by a pilot of terrain marking points. By night it can cause the unreliability of safety due a pilot's loss of orientation.
 - Engine stoppage in the air. At a defined flight altitude a pilot can still start the engine and continue the flight, i.e. the efficiency unreliability occurs. However, if the altitude is insufficient, or the pilot fails to restart the engine, the safety unreliability can occur.

Figures 1a and 1b show the above described object models.

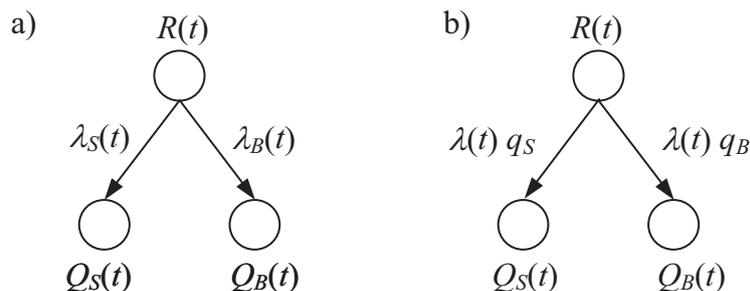


Fig. 1. An object model with a damage: with a result to description 2.1 (a), with a result to description 2.2. (b)

where:

- $\lambda_S(t)$ - severity of damage to this part of an object, the damage to which results in the efficiency unreliability,
- $\lambda_B(t)$ - severity of damage to this part of an object, the damage to which results in the safety unreliability,
- $\lambda(t)$ - object damage severity,
- q_S - probability of an occurrence of efficiency unreliability occurrence in a damaged object,
- q_B - probability of an occurrence of safety unreliability in a damaged object,
- $R(t)$ - probability of a failure-free functioning of an object,
- $Q_S(t)$ - efficiency unreliability,
- $Q_B(t)$ - safety unreliability.

The 1a graph can be described by the Kolmogorov-Chapman system of differential equations:

$$\begin{aligned}
 R'(t) &= -[\lambda_S(t) + \lambda_B(t)] R(t), \\
 Q'_S(t) &= \lambda_S(t) R(t), \\
 Q'_B(t) &= \lambda_B(t) R(t), \\
 R(t) + Q_S(t) + Q_B(t) &= 1, \\
 R(0) = 1, \quad Q_B(0) = Q_S(0) &= 0.
 \end{aligned} \tag{1}$$

By solving this system of differential equations (1) we obtain:

$$\begin{aligned}
 R(t) &= \exp\{-[\Lambda_S(t) + \Lambda_B(t)]\} = R_S(t) R_B(t), \\
 R(t) &= \exp\{-[\Lambda_S(t) + \Lambda_B(t)]\} = R_S(t) R_B(t), \\
 Q_S(t) &= \int_0^t \lambda_S(\tau) R(\tau) d\tau, \\
 Q_B(t) &= \int_0^t \lambda_B(\tau) R(\tau) d\tau.
 \end{aligned} \tag{2}$$

If we assume $\lambda_S(t) = \lambda_S$, $\lambda_B(t) = \lambda_B$, the formula (2) transforms into:

$$\begin{aligned}
 R(t) &= \exp[-(\lambda_S + \lambda_B)t], \\
 Q_S(t) &= \frac{\lambda_S}{\lambda_S + \lambda_B} [1 - R(t)], \\
 Q_B(t) &= \frac{\lambda_B}{\lambda_S + \lambda_B} [1 - R(t)].
 \end{aligned} \tag{3}$$

From the obtained solution we obtain the following relations:

$$\begin{aligned}
 R(t) &= \exp[-\Lambda_S(t)] \cdot \exp[-\Lambda_B(t)] = R_S(t) \cdot R_B(t), \\
 Q_B(t) &= 1 - R(t) - Q_S(t) = Q(t) - Q_S(t).
 \end{aligned} \tag{4}$$

A historical probability $R_S(t)$ was called the efficiency reliability, and the probability $R_B(t)$ the reliability of safety, what was criticized by some specialist, e.g. by Prof. Zbigniew Smalko.

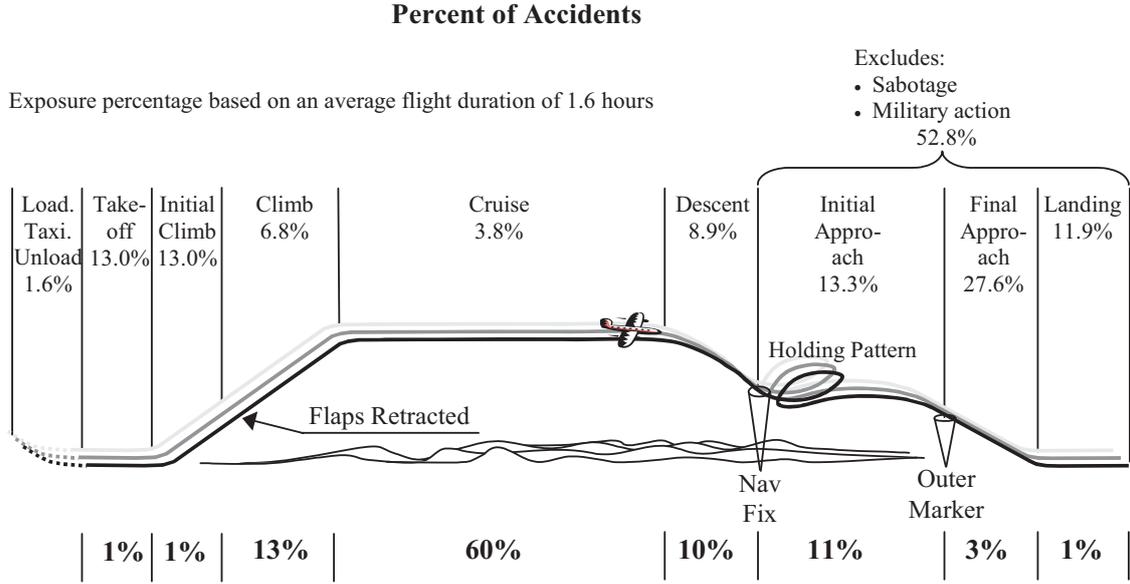


Fig. 1b. Diagram presenting the flight time

The model in Fig. 1 b can be described by the following Kolmogorov-Chapman system of differential equations:

$$\begin{aligned}
 R'(t) &= -\lambda(t) R(t), \\
 Q'_S(t) &= q_S \lambda(t) R(t), \\
 Q'_B(t) &= q_B \lambda(t) R(t).
 \end{aligned} \tag{5}$$

Solving this system (5) we obtain:

$$\begin{aligned}
 R(t) &= \exp\left[-\int_0^t \lambda(\tau) d\tau\right], \\
 Q_S(t) &= \int_0^t \lambda(\tau) q_S R(\tau) d\tau = q_S \int_0^t \lambda(\tau) R(\tau) d\tau = Q(t) q_S, \\
 Q_B(t) &= q_B Q(t).
 \end{aligned} \tag{6}$$

$Q(t)$ - probability of damage to an object.

$$R(t) = \exp[-q_S \lambda(t) - q_B \lambda(t)] = R_S(t) R_B(t).$$

An example of the object shown in Fig. 1 b is the object presented in Fig. 2.

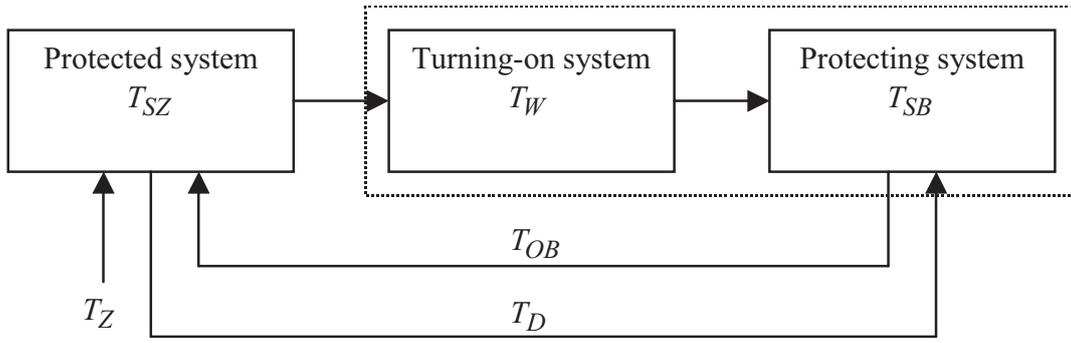


Fig. 2. Elementary model of a system "protected system – turning on system – protecting system"

The object consists of a protected system (e.g. the drive of an aircraft), a protecting system (e.g. the device to start an engine in the air), and a turning-on system (it can be a turning-on pilot to turn on a device to start engine in the air). The work duration of a protected object till there is a need for the protecting device to act is T_{SZ} with a distribution function $F_{SZ}(t)$ (e.g. the aircraft flight duration till the engine stops in the air). When there is such a need, the protecting device turns on. The duration of the turn-on action is T_W with time distribution function $F_W(t)$ (In case the engine stalls in the air, this time depends on a time to ascertain the engine has stopped, to analyse the situation, to take a decision, to execute this decision, to check if the executed decision was effective, etc.) The working time of the protecting device till damage is T_{SB} with a distribution function $F_{SB}(t)$. If a protecting device is fit it starts to counteract a dangerous situation (e.g. restarting the engine in the air). However, if such a device is unfit, the safety unreliability occurs.

The time of the counteraction countering the dangerous situation is T_{OB} , with a distribution function $F_{OB}(t)$. The time of a counteraction of a dangerous situation is limited by the available time T_D , with the distribution function $F_D(t)$ (e.g. the time to restart the engine depending on the flight altitude). If the effect of a counteraction was successful, we say that the efficiency unreliability occurred (an aircraft interrupts its task flight and returns to a base).

Shall we, for the model under discussion, assume that the turn-on time of the protecting device is 0, the counteraction time is 0 and the available time is infinite, then for the model shown in Fig. 1 b:

$$q_S = P(T_{SZ} < T_{SB}) \text{ and } q_B = P(T_{SZ} > T_{SB}), \quad (7)$$

$$q_S = P(T_{SZ} < T_{SB}) = \int_0^{\infty} F_{SZ}(t) dF_{SB}(t).$$

If the random variables T_{SZ} and T_{SB} have exponential distribution correspondingly of an intensity λ_{SZ} and λ_{SB} , the solution of the system of equations (5) will be:

$$Q_S(t) = \frac{\lambda_{SB}}{\lambda_{SB} + \lambda_{SZ}} [1 - R(t)] = q_S Q(t), \quad (8)$$

$$Q_B(t) = \frac{\lambda_{SZ}}{\lambda_{SB} + \lambda_{SZ}} [1 - R(t)] = q_B Q(t).$$

We will determine the asymptotical values of the efficiency unreliability and safety unreliability from formulas:

$$Q_S = \lim_{t \rightarrow \infty} Q_S(t) = q_S, \quad (9)$$

$$Q_B = \lim_{t \rightarrow \infty} Q_B(t) = q_B. \quad (10)$$

For a full analysis of the presented model see dissertations:

- Jan Borgoń: *Niezawodność i bezpieczeństwo systemu pilot – statek powietrzny* (Reliability and safety of the pilot – aircraft system) WAT, Warszawa 1985,
- Józef Żurek: *Modelowania symboliczne systemów bezpieczeństwa i niezawodności w transporcie lotniczym* (Symbolic models of safety and reliability systems in air transport), Wydział Transportu Politechniki Warszawskiej, Warszawa 1998.

3. Serial structure object consisting of three-state elements

Fig. 3 shows a serial structure model consisting of three-state elements.

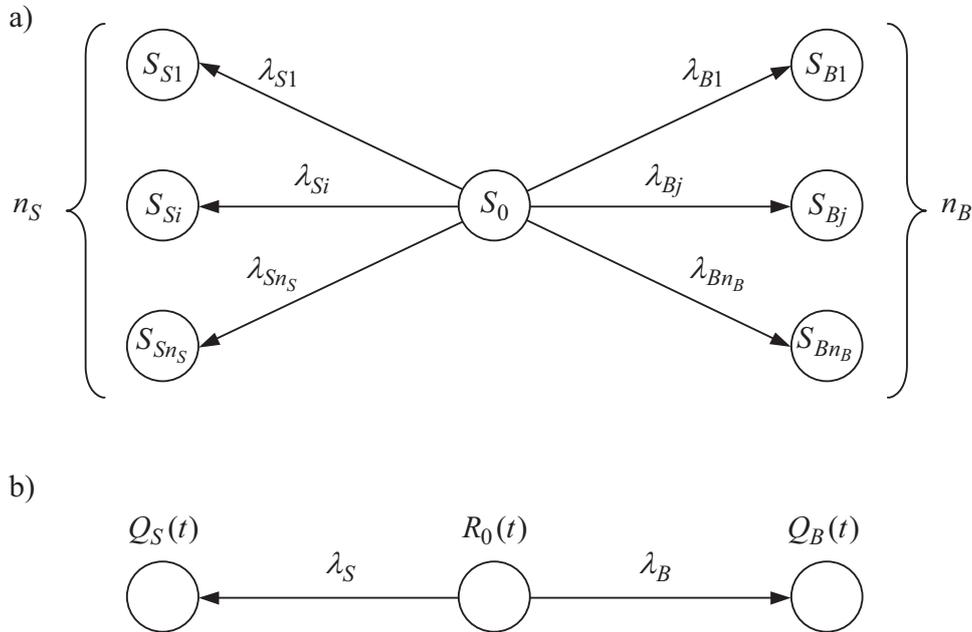


Fig. 3. Model of a three-state renewable object of a reversible state of efficiency unreliability (a), a reduced model (b)

The graph is described by the Kolmogorov-Chapman system of differential equations (1), where:

$$\lambda_S = \sum_{i=1}^{n_S} \lambda_{Si}, \quad \lambda_B = \sum_{j=1}^{n_B} \lambda_{Bj}.$$

By solving the system of differential equations we will obtain specific coefficients of the object reliability and safety:

$$Q_S(t) = \sum_{i=1}^{n_S} Q_{Si}(t), \quad (11)$$

$$\begin{aligned}
 Q_B(t) &= \sum_{j=1}^{n_B} Q_{Bj}(t), \\
 R(t) &= R_S(t)R_B(t), \\
 R_S(t) &= \exp[-\lambda_S t], \\
 R_B(t) &= \exp[-\lambda_B t].
 \end{aligned} \tag{12}$$

4. Three-state renewable object

In three-state renewable objects the state of the efficiency unreliability is reversible, this is illustrated in Fig. 4.

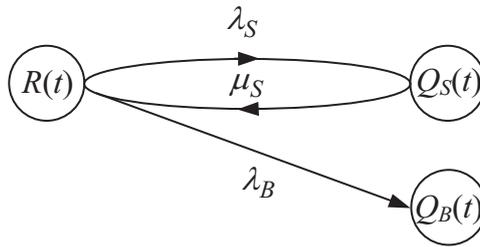


Fig. 4. Graph of a three-state renewable object

The Kolmogorov-Chapman differential equation has the following form:

$$\begin{aligned}
 R'(t) &= -(\lambda_S + \lambda_B)R + \mu_S Q_S, \\
 Q_S'(t) &= -\mu_S Q_S(t) + \lambda_S R(t), \\
 Q_B'(t) &= \lambda_B R(t) \quad (13), \\
 R(0) &= 1, \quad Q_S(0) = Q_B(0) = 0, \\
 R(t) + Q_S(t) + Q_B(t) &= 1.
 \end{aligned}$$

Solving the equations system we obtain:

$$\begin{aligned}
 R(t) &= \frac{1}{C} e^{x_1 t} \left[(x_1 + \mu_S) - (x_2 + \mu_S) e^{-Ct} \right], \\
 Q_S(t) &= \frac{\lambda_S}{C} e^{x_1 t} \left(1 - e^{-Ct} \right), \\
 Q_B(t) &= 1 - Q_S(t) - R(t),
 \end{aligned} \tag{14}$$

where:

$$x_{1/2} = \frac{1}{2}(-A \pm C),$$

$$C = \sqrt{A^2 - 4B},$$

$$A = \lambda_B + \lambda_S + \mu_S.$$

The average working time of an object till the safety unreliability occurring, is given by the formula:

$$\bar{T}_B = \frac{1}{\lambda_B} \left(1 + \frac{\lambda_S}{\mu_S} \right). \quad (15)$$

Fig.5 shows the dependence of the expected working time of a system till an occurrence of the state of the safety unreliability on the renewal intensity. When the renewal intensity decreases to zero, the T_B time rises to infinity. When the renewal intensity grows, the time T_B acquires a constant value. It is a trivial conclusion, since if a system is continuously under repair and does not work, it is safe.

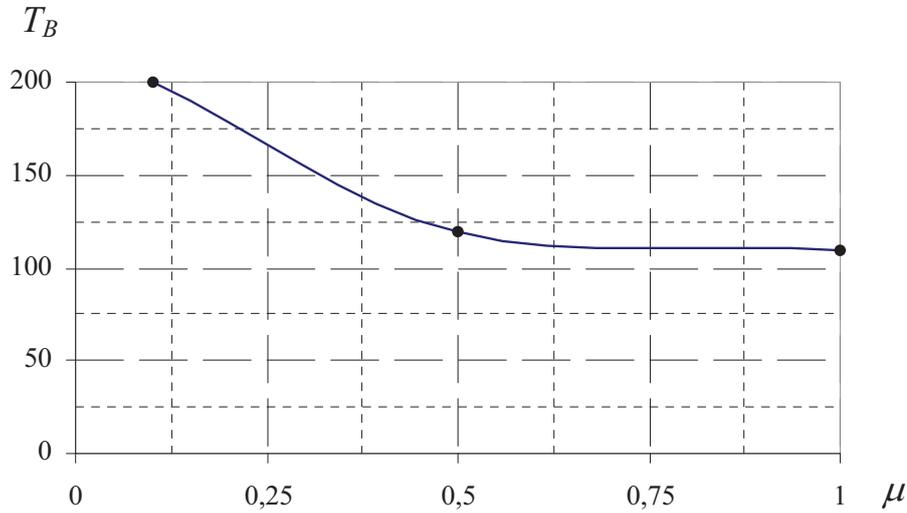


Fig.5. Relation $T_B = f(\mu)$ for a renewable three-state system

The intensity of the safety unreliability is given by:

$$\lambda_B(t) = \frac{Q'_B(t)}{R(t) + Q_S(t)} = \frac{\lambda_B R(t)}{R(t) + Q_S(t)}. \quad (16)$$

Formulas for asymptotical values of the safety unreliability:

$$\lambda_B(0) = \lambda_B$$

$$\lambda_B(\infty) = \lim_{t \rightarrow \infty} \lambda_B(t) = \frac{(x_1 + \mu_S) \lambda_B}{x_1 + \mu_S + \lambda_S}. \quad (17)$$

From the above we conclude the safety unreliability intensity is a decreasing function.

A three-state renewable object, where in a renewable state the safety unreliability of the $\lambda_{SB}(t)$ intensity can occur, is presented in Fig. 6.

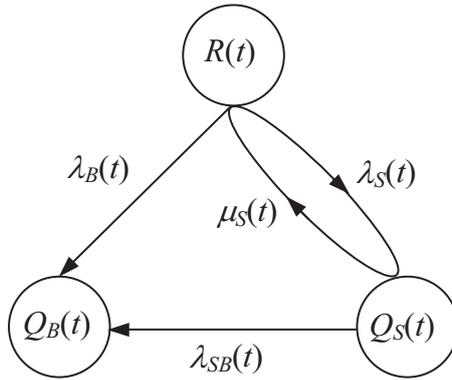


Fig. 6. Graph of a three-state system with a dangerous renewal state

The system working time until it converts into a state of the safety unreliability is given by the formula:

$$T_B = \frac{\mu + \lambda_{SB} + \lambda_S}{\mu \lambda_B + \lambda_{SB} \lambda_S + \lambda_{SB} \lambda_B}. \quad (18)$$

Fig. 7 presents the dependence of an average working time till a safety unreliability occurs on the intensity function of a transition, from the efficiency unreliability state to the state of the safety unreliability for parameters: $\mu = 1$; $\lambda_S = 0,1$; $\lambda_B = 0,01$.

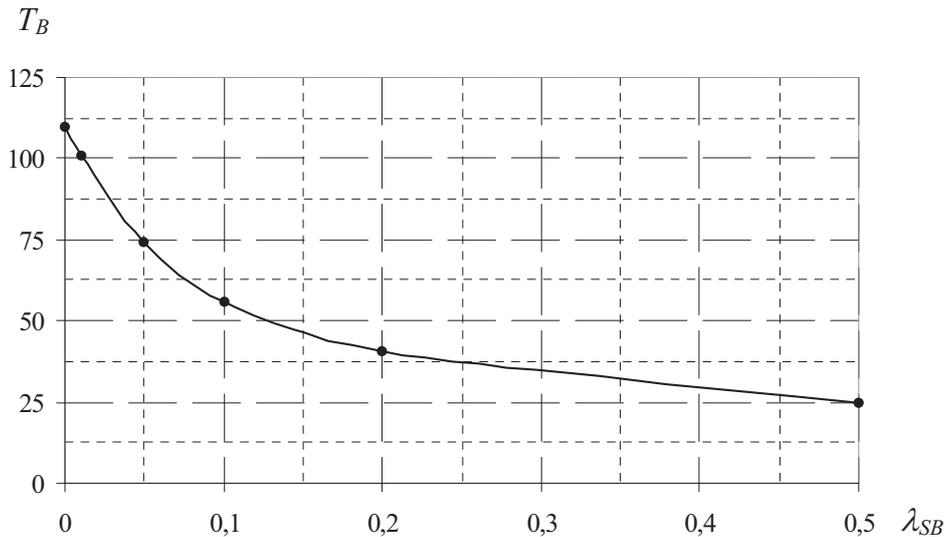


Fig. 7. Graph: Relationship $T_B = f(\lambda_{SB})$ for a renewable three-state system

When in state of renewal the safety unreliability intensity grows, the system safety unreliability decreases. When compared to the above-discussed system, where the renewal system was safe, it becomes obvious.

5. Four-state object

A four-state object was described by Prof. Jerzy Jaźwiński and Prof. Krystyna Ważyńska-Fiok in Zagadnienia Eksploatacji Maszyn (Problems of Machine Operation), book z. 4, 1989 and book z.1, 1991. Fig.8 shows a graph of such an object.

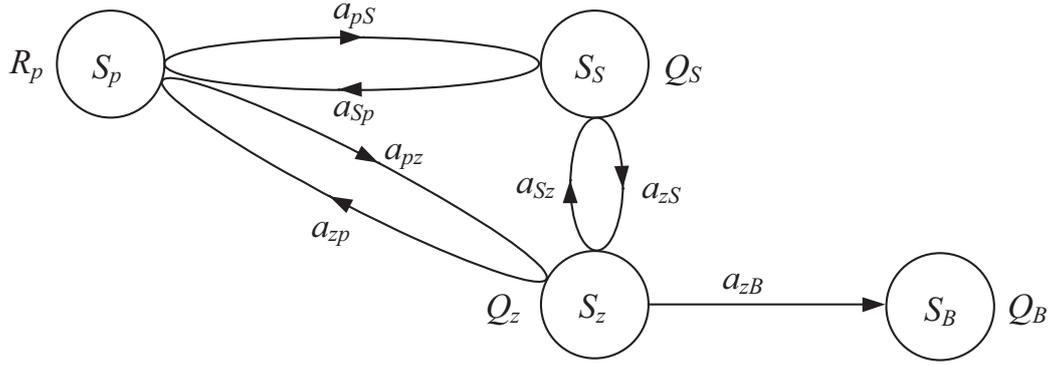


Fig. 8. Graph of a four-state object

Denotations in Fig. 8: S_p - safety state, S_z - threatened state, S_s - efficiency unreliability state, S_B - safety unreliability state, R_p, Q_z, Q_s, Q_B - state probabilities, $a_{\alpha, \beta}$ - transition intensities

The graph in Fig. 8 can be described by Kolmogorov-Chapman system of differential equations:

$$\begin{aligned}
 R_p' &= -(Q_{pS} + Q_{pz})R_p + a_{Sp} Q_S + a_{zp} Q_z, \\
 Q_S' &= a_{pS} R_p + a_{zS} Q_z - (a_{Sp} + a_{Sz})Q_S, \\
 Q_z' &= a_{pz} R_p + a_{Sz} Q_S - (a_{zp} + a_{zS} + Q_{zB})Q_z, \\
 Q_B' &= a_{zB} Q_z
 \end{aligned} \tag{19}$$

$$R_p + Q_z + Q_S + Q_B = 1,$$

$$R_p(0) = 1,$$

$$Q_z(0) = Q_S(0) = Q_B(0) = 0.$$

The value of the latency time and the variance of the system operation time till a transition into the state of the safety unreliability are described by the following formulas:

$$E[T_B] = \frac{D - a_{pz} a_{zB}}{C}, \tag{20}$$

$$V[T_B] = \frac{2D}{C} E[T_B] - \{E[T_B]\}^2 - \frac{2A}{C}, \tag{21}$$

where:

$$\begin{aligned}
 A &= a_{pz} + a_{pS} + a_{zB} + a_{Sp} + a_{zS} + a_{zp} + a_{Sz}, \\
 D &= a_{pS} (a_{zp} + a_{zB} + a_{Sz} + a_{zS}) + a_{Sz} (a_{zp} + a_{pz}) + a_{Sp} (a_{zp} + a_{pz} + a_{zS}) + a_{zB} \\
 &\quad (a_{pz} + a_{Sp} + a_{Sz}) + a_{pz} a_{zS}, \\
 C &= a_{zB} [a_{pz} a_{Sp} + a_{Sz} (a_{pz} + a_{pS})].
 \end{aligned}$$

Let's consider two particular cases of a system presented in the graph (Fig.8).

1) A four-state renewable system with an intermediate endangered safety state is shown in Fig.9.

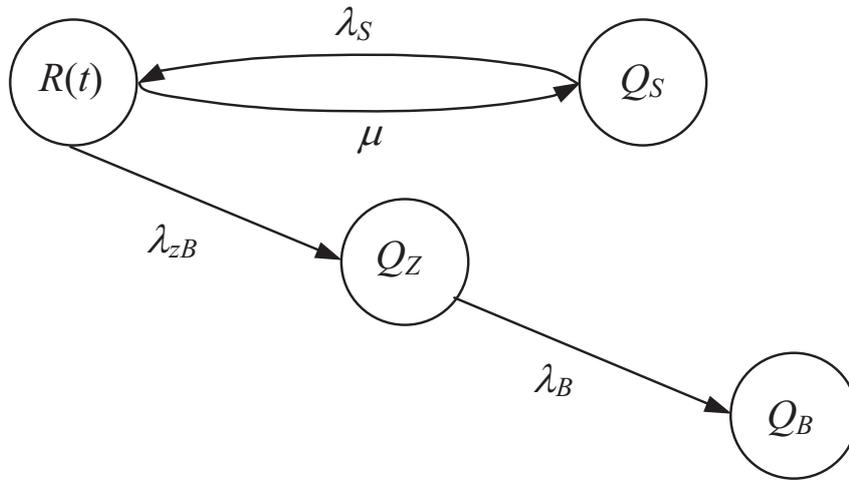


Fig. 9. A graph of a four-state renewable system with an intermediate endangered safety state

The waiting time of the system operation till safety unreliability occurs is given by:

$$T_B = \frac{1}{\lambda_{zB}} + \frac{1}{\lambda_S} + \frac{\lambda_S}{\mu \lambda_{zB}}. \quad (22)$$

Fig. 10 shows the dependence of the system operation time till an occurrence of the safety unreliability on the renewal intensity for $\lambda_B = 0,01$, $\lambda_S = 0,1$, $\lambda_{zB} = 0,1$.

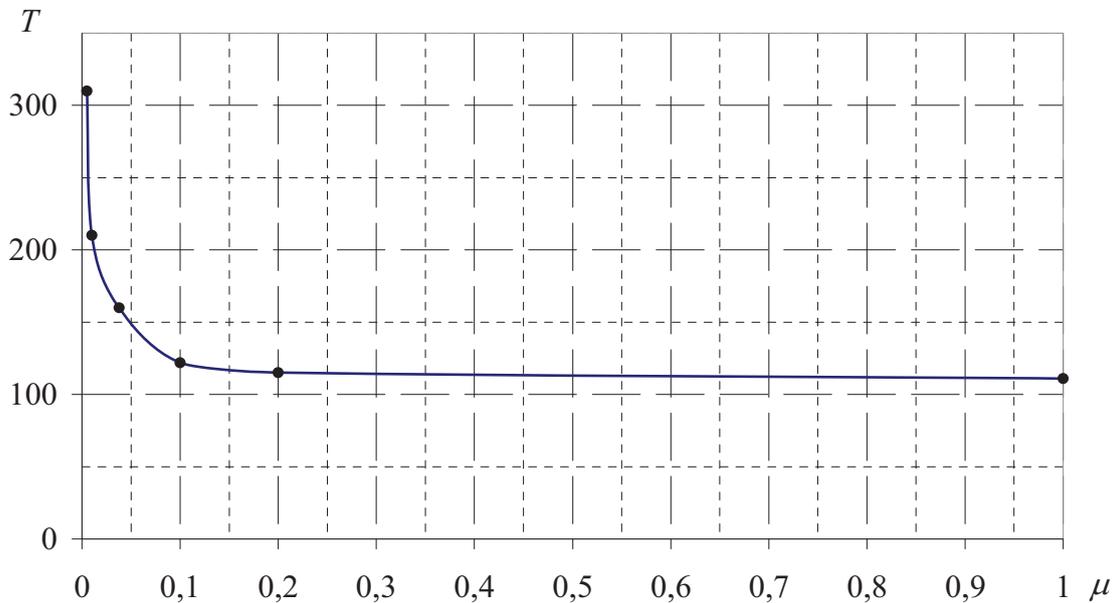


Fig. 10. Graph of the relationship $T_B = f(\mu)$ for a renewable three-state system

2) A graph of a four-state renewable system for which a counteraction against a danger restoring to the renewable state is possible is shown in Fig. 11.

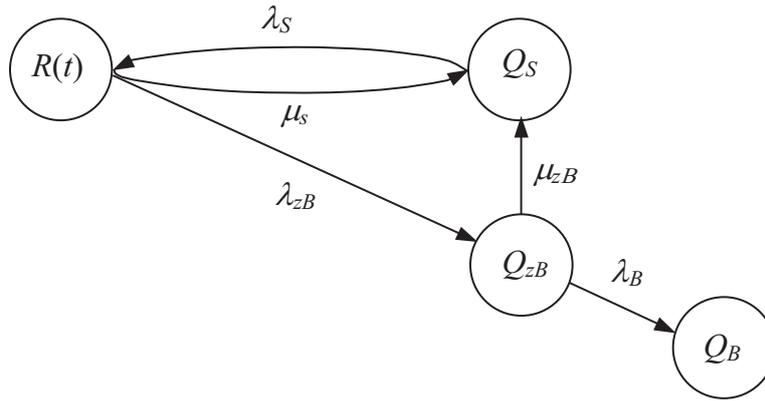


Fig. 11. A graph of a four-state renewable system for which a counteraction against a danger restoring to the renewable state is possible

The expected value of the system operation time till safety unreliability occurs can be calculated from a formula:

$$T_B = \frac{\mu_{zB}}{\lambda_{zB}\lambda_B} + \frac{1}{\lambda_{zB}} + \frac{1}{\lambda_B} + \frac{\lambda_S\mu_{zB}}{\lambda_B\lambda_{zB}\lambda_S} + \frac{\lambda_S}{\lambda_{zB}\mu_S} + \frac{\mu_{zB}}{\mu_S\lambda_B}. \quad (23)$$

Example:

for data $\mu_S = 1; \lambda_{zB} = 0,1; \lambda_B = 0,01; \mu_{zB} = 0,1 - T_B = 321,$

for $\mu_{zB} = 0 - T_B = 110.$

It follows that the intensity of a system transition from a state of endangered safety to the state of renewal significantly increases the safe operation time of a system.

6. Object realizing a complex intentional programme

An intentional state is a state complying with our intention. A set of intentional states creates an intentional programme. Examples of intentional states: a train timetable, a flight schedule or different transport tasks. A simplest intentional program is a two state program: "operation – waiting for operation", "signal – road free – signal – road occupied" In every intentional program damages followed by different results can occur. This signifies that a program executing a complex intentional program can take different unreliability-safety states (Fig. 12).

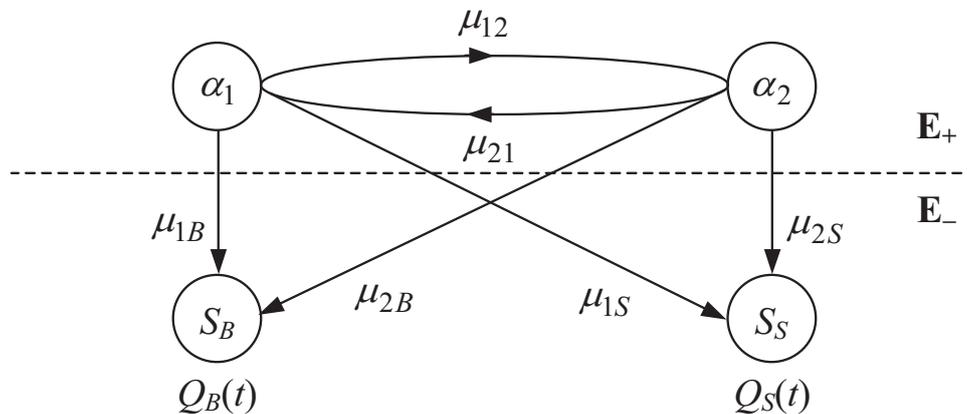


Fig. 12. Model of a system with two reversible intentional states

where:

- α_1 and α_2 - symbols of the first and second intentional state,
- $F_{12}(t)$ - distribution functions of a dwell time on intentional state α_2 on condition of a transition into an intentional state α_2 ,
- $F_{21}(t)$ - distribution functions of an intentional state duration α_2 on condition of transition into an intentional state α_1 ,
- μ_{1B} and μ_{2B} - correspondingly: intensities of a transition from intentional states α_1 and α_2 into a state of safety unreliability S_B ,
- μ_{1S} and μ_{2S} - correspondingly: intensities of a transition from intentional states α_1 and α_2 into a state of efficiency unreliability S_B .

If we analyse the system as presented in Fig. 12 using a half-Markov's processes we can determine that value of an expected dwell time in a subset of intentional states α_1 and α_2 (subset E_+).

The following formula determines the expected value of time $\bar{\tau}$:

$$\bar{\tau}_1 = \frac{\frac{1}{\mu_1} [1 - \tilde{F}_{12}(\mu_1)] + \frac{1}{\mu_2} [1 - \tilde{F}_{21}(\mu_2) \tilde{F}_{12}(\mu_1)]}{1 - \tilde{F}_{12}(\mu_1) \tilde{F}_{21}(\mu_2)}, \quad (24)$$

where:

- $\tilde{F}_{12}(\mu_1)$ - Laplace-Stieltjes transformation of the $F_{12}(t)$ time function for an μ_1 argument,
- $\tilde{F}_{21}(\mu_2)$ - Laplace-Stieltjes transformation of the $F_{21}(t)$ time function for an μ_2 argument.

Let's consider an elementary example when the dwell time of an object in an intentional state shows a single point distribution with a distribution function:

$$F_{ij}(t) = \begin{cases} 1 & \text{dla } t \geq L_i \\ 0 & \text{dla } t < L_i \end{cases} \text{ for } i, j = 1, 2. \quad (25)$$

With a single-point distribution function $F_{ij}(t)$, the Laplacea-Stieltjesa having a random variable with w single-point distribution takes the form of:

$$\tilde{F}_{12}(s) = \exp[-s L_1], \quad \tilde{F}_{21}(s) = \exp[-s L_{21}] \quad (26)$$

If we substitute (26) into (24) we obtain the expected value of the object dwell time in the subset of times E_+ :

$$\bar{\tau}_1 = \frac{\frac{1}{\mu_1} + \exp[-\mu_1 L_1] \left[\frac{1}{\mu_2} - \frac{1}{\mu_1} \right] - \frac{1}{\mu_2} \exp[-(\mu_1 L_1 + \mu_2 L_2)]}{1 - \exp[-(\mu_1 L_1 + \mu_2 L_2)]}. \quad (27)$$

An example of a complex intentional programme is a flight task shown in Fig. 13.

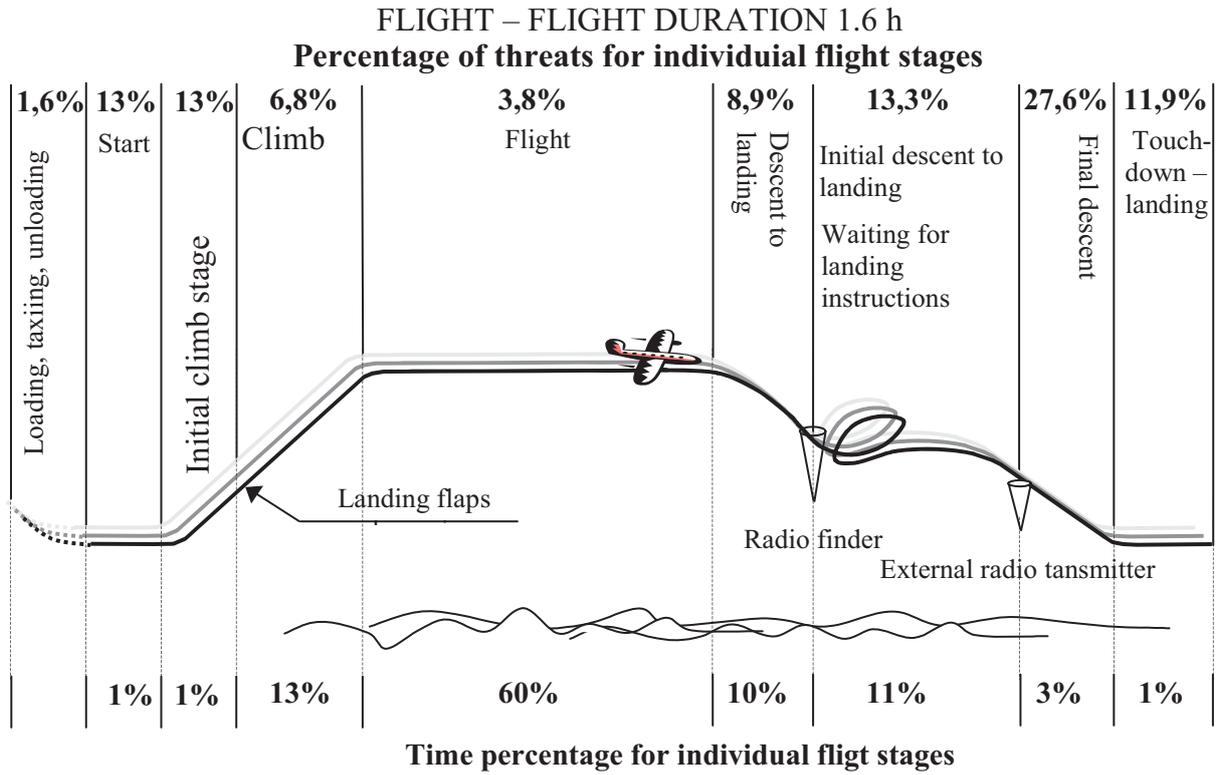


Fig. 13. Portions of individual flight stages in a 1.6 hour flight and percentage of threats for individual flight stages

A model of this flight task is a graph presented in Fig. 14.

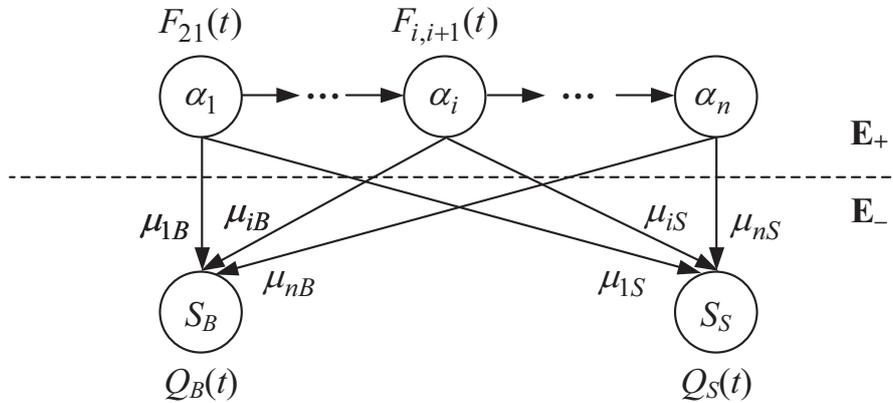


Fig. 14. Model of a system of a complex non-cyclic intentional programme

Analysing the graph shown in Fig. 14 with the help of half-Markov's process we obtain the formula for the expected value of the object dwell time in a subset of intentional states E_+ :

$$\bar{\tau}_1 = \sum_{i=1}^n \frac{1}{\mu_i} [1 - \tilde{F}(\mu_i)] \prod_{j=1}^{i-1} F_{j,j+1}(\mu_j). \quad (28)$$

By substitution of the (26) formula into the (28) formula we obtain the expected dwell time in a subset of intentional states E_+ , on condition that the dwell time in the i -th intentional state has a 1-point distribution.

$$\bar{\tau}_1 = \sum_{i=1}^n \frac{1}{\mu_i} \left[1 - e^{-\mu_i L_i} \right] e^{-\sum_{j=1}^{i-1} \mu_j L_j} \quad (29)$$

The model presented in Fig. 12 was many a times used, e.g. a cover of a book by Prof. Krystyna Ważyńska-Fiok and Prof. Jerzy Jaźwiński "Reliability of Technical Systems", PWN, Warszawa 1990, stamp at the VI Symposium for Systems Safety, post stamp at various symposiums, ex-libris of Prof. Krystyna Ważyńska-Fiok and Prof. Jerzy Jaźwiński.

The model shown in Fig. 14 was described in many publications, particularly in a book by Prof. Jerzy Jaźwiński and Prof. Franciszek Grabski "Niektóre problemy modelowania systemów transportowych" ("Some Problems of Modelling of Transport Systems"), published by the Instytut Technologii Eksploatacji (the Institute of Operation Technology), Radom 2003.

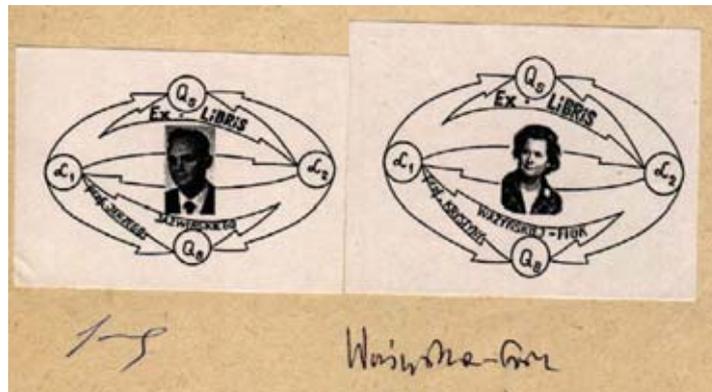
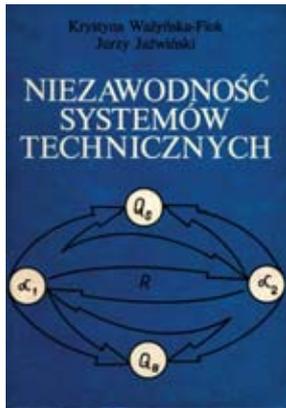


Fig. 15. Different illustrations and uses of the model presented in Fig. 12

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